

Orange Public Schools

Office of Curriculum & Instruction
2019-2020 Mathematics Curriculum Guide



Second Grade Mathematics

Unit 4:

Eureka - Module 4: Add/Subtract Within 200
(Word Problems up to 100)

December 2, 2019 – January 31, 2020

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Yearlong Pacing Guide: Second Grade

<i>Eureka Math</i>	<i>Eureka Module Standards</i>
Unit 1- Module 1: Sum/ Difference to 20 Sept 9- Sept 27	2OA1, 2OA2, 2NBT5
Unit 2-Module 2: Add/ Subtract Length Sept 30- Oct 18	2MD1, 2MD2, 2MD3, 2MD4, 2MD5,2MD6
Unit 3- Module 3: Place Value, Count to 1000 Oct 21 – Nov 26	2NBT1, 2NBT2, 2NBT3, 2NBT4
Unit 4- Module 4: Add/ Subtract within 200 (Word Problems up to 100) Dec 2- Jan 31	2OA1, 2NBT5, 2NBT6, 2NBT7, 2NBT8, 2NBT9
Unit 5- Module 5: Add/ Subtract within 1000 (Word Problems up to 100) Feb 3- March 13	2NBT7, 2NBT8, 2NBT9
Unit 6- Module 7: Length, Money, Data March 16- May 8	2NBT5, 2MD1, 2MD2, 2MD3, 2MD4, 2MD5, 2MD6, 2MD8, 2MD9, 2MD10
Unit 7- Module 8: Time, Shapes, Fractions May 11- June 5	2MD7, 2G1, 2G3
Unit 8- Module 6: Foundations Multiplication Division June 8- EOSY	2OA3, 2OA4, 2G2

References

“Eureka Math” *Gt Minds*. 2018 < <https://greatminds.org/account/products>

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Module 4

Essential Questions	Enduring Understandings
<ul style="list-style-type: none"> • How does the position of a digit in a number affect its value? • In what different ways can numbers be grouped? • What are strategies to make a reasonable estimate? • How do operations affect numbers? • How is an equation like a balance scale? • How do you use addition and subtraction to solve real world problems? • How can we compare and contrast numbers? • How can memorizing the basic addition and subtraction facts help me? • How does understanding place value help you solve double digit addition and subtraction problems? 	<ul style="list-style-type: none"> • The position of a digit in a number determines its value. • The groupings of ones, tens and hundreds for a given number can be taken apart in different ways. Estimation is a strategy for getting as close as possible to an exact answer. • The significance of numbers affects the outcome of operations on them. • The totals on each side of an equal sign equal each other, similar to that of a balance scale. Real-life situations regarding the increase or decrease of numbers/objects can be applied to addition and subtraction. • The overall value of a number is determined by its location within a number. • Two and three digit numbers can be compared based on the meaning of the hundreds, tens, and ones digits using the $>$, $<$, and $=$ symbols to record the results of the comparisons • The position of a digit in a number determines its value. • The groupings of ones, tens and hundreds for a given number can be taken apart in different ways. • Knowledge of addition and subtraction facts makes the computation of larger numbers easier to solve. • Numbers can be aligned in accordance with place value to correctly add denominations of ten. Flexible methods of computation involve grouping finding sums and differences? numbers in strategic ways. Mental math strategies can be used to solve problems involving numbers.

Performance Overview

- Topic A's strategies lead naturally to work with the algorithms for addition (Topic B) and subtraction (Topic C). In these two topics, students represent place value strategies with place value disks and math drawings. Students work with composing 1 ten from 10 ones or decomposing 1 ten as 10 ones (with minuends within 100). After the Mid-Module Assessment, students continue working with manipulatives and math drawings to make sense of problems in which they compose or decompose twice. Topic D focuses on addition, with the new complexity of composing 1 hundred from 10 tens within 200 in problems with up to four addends. Subtraction in Topic E involves subtracting when decomposing 1 hundred for 10 tens and 1 ten for 10 ones.
- Throughout the module, manipulatives and math drawings allow students to see numbers in terms of place value units and serve as a reminder that students must add like units (e.g., knowing that $74 + 38$ is 7 tens + 3 tens and 4 ones + 8 ones). The focus is often on computational strategies with bare numbers (i.e., no context) so that total attention is given to understanding the value of each digit within a number, as well as why the algorithm works. Students use the place value chart as an organizer. Simultaneous use of a vertical form and a place value chart allows students to better recognize both the value of numbers when they are not on the place value chart and like units. The same is true when students make math drawings and use place value language to relate each step of the drawing to the vertical form. The different representations serve to solidify the understanding of the composition and decomposition of units, moving from concrete to pictorial to abstract. Throughout the work, students are encouraged to explain their actions and analyses and to use the relationship between addition and subtraction to check their work.

Module 4: Addition and Subtraction within 200 with Word Problems to 100**Pacing:**

December 2, 2019– January 31, 2020

Suggested Instructional Days: 35

Topic	Lesson	Student Lesson Objective/ Supportive Videos
Topic A: Sums and Differences within 100	Lesson 1	Relate 1 more, 1 less, 10 more, and 10 less to addition and subtraction of 1 and 10. https://www.youtube.com/watch?v=IB2mHoinybw&list=PLvolZqLMhJmkb4rXaDraEy4I-TLolZNTM
	Lesson 2	Add and subtract multiples of 10 including counting on to subtract https://www.youtube.com/watch?v=au8_fsJwS-I&list=PLvolZqLMhJmkb4rXaDraEy4I-TLolZNTM&index=2
	Lesson 3 & 4	Add and subtract multiples of 10 and some ones within 100 https://www.youtube.com/watch?v=mdi5Gk8HVWk&index=3&list=PLvolZqLMhJmkb4rXaDraEy4I-TLolZNTM https://www.youtube.com/watch?v=0yckkOhgtXU&index=4&list=PLvolZqLMhJmkb4rXaDraEy4I-TLolZNTM
	Lesson 5	Solve one- and two-step word problems within 100 using strategies based on place value. https://www.youtube.com/watch?v=8pPZXUIw5aU&list=PLvolZqLMhJmkb4rXaDraEy4I-TLolZNTM&index=5
Topic B: Strategies for Composing a Ten	Lesson 6	Use manipulatives to represent the composition of 10 ones as 1 ten with two-digit addends https://www.youtube.com/watch?v=urYVOIAlltY&index=6&list=PLvolZqLMhJmkb4rXaDraEy4I-TLolZNTM
	Lesson 7	Relate addition using manipulatives to a written vertical method https://www.youtube.com/watch?v=iJH5uN33Sbc&index=7&list=PLvolZqLMhJmkb4rXaDraEy4I-TLolZNTM
	Lesson 8	Use math drawings to represent the composition and relate drawings to a written method https://www.youtube.com/watch?v=E_sAOB0UIgE&index=8&list=PLvolZqLMhJmkb4rXaDraEy4I-TLolZNTM
	Lesson 9 & 10	Use math drawings to represent the composition when adding a two-digit to a three-digit addend https://www.youtube.com/watch?v=htHhMUOlkgQ&index=9&list=PLvolZqLMhJmkb4rXaDraEy4I-TLolZNTM https://www.youtube.com/watch?v=GVouDyY5QxI&list=PLvolZqLMhJmkb4rXaDraEy4I-TLolZNTM&index=10

<p>Topic C: Strategies for Decomposing a Ten</p>	Lesson 11	<p>Represent subtraction with and without the decomposition of 1 ten as 10 ones with manipulatives. https://www.youtube.com/watch?v=XMdbStwv8QI&list=PLvolZqLMhJmkb4rXaDraEy4I-TLolZNTM&index=11</p>
	Lesson 12	<p>Relate manipulative representations to a written method https://www.youtube.com/watch?v=96-k1P-aNEk&index=12&list=PLvolZqLMhJmkb4rXaDraEy4I-TLolZNTM</p>
	Lesson 13	<p>Use math drawings to represent subtraction with and without decomposition and relate drawings to a written method https://www.youtube.com/watch?v=df3YAB4psXM&index=13&list=PLvolZqLMhJmkb4rXaDraEy4I-TLolZNTM</p>
	Lesson 14 & 15	<p>Represent subtraction with and without the decomposition when there is a three-digit minuend https://www.youtube.com/watch?v=C_LeUa5QF5o&list=PLvolZqLMhJmkb4rXaDraEy4I-TLolZNTM&index=14 https://www.youtube.com/watch?v=4qaPt4WBADQ&list=PLvolZqLMhJmkb4rXaDraEy4I-TLolZNTM&index=15</p>
	Lesson 16	<p>Solve one- and two-step word problems within 100 using strategies based on place value. https://www.youtube.com/watch?v=sinn8f8p778&list=PLvolZqLMhJmkb4rXaDraEy4I-TLolZNTM&index=16</p>
Mid-Module Assessment Task		
<p>Topic D: Strategies for Composing Tens and Hundreds</p>	Lesson 17	<p>Use mental strategies to relate compositions of 10 tens as 1 hundred to 10 ones as 1 ten https://www.youtube.com/watch?v=0oLo0L3KRrM&list=PLvolZqLMhJmkb4rXaDraEy4I-TLolZNTM&index=17</p>
	Lesson 18	<p>Use manipulatives to represent addition with two compositions https://www.youtube.com/watch?v=mLfahhgWSno&index=18&list=PLvolZqLMhJmkb4rXaDraEy4I-TLolZNTM</p>
	Lesson 19	<p>Relate manipulative representations to a written method https://www.youtube.com/watch?v=jKJZJg-F2h8&index=19&list=PLvolZqLMhJmkb4rXaDraEy4I-TLolZNTM</p>
	Lesson 20 & 21	<p>Use math drawings to represent additions with up to two compositions and relate drawings to a written method https://www.youtube.com/watch?v=lbLdKFVpzms&list=PLvolZqLMhJmkb4rXaDraEy4I-TLolZNTM&index=20 https://www.youtube.com/watch?v=yXKabm5UW5Y&list=PLvolZqLMhJmkb4rXaDraEy4I-TLolZNTM&index=21</p>
	Lesson 22	<p>Solve additions with up to four addends with totals within 200 with and without two compositions of larger units https://www.youtube.com/watch?v=fkuxYeZf40U&list=PLvolZqLMhJmkb4rXaDraEy4I-TLolZNTM&index=22</p>

<p>Topic E: Strategies for Decomposing Tens and Hundreds</p>	Lesson 23	Use number bonds to break apart three-digit minuends and subtract from the hundred. https://www.youtube.com/watch?v=XI8oH45j4_0&list=PLvolZqLMhJmkb4rXaDraEy4I-TLolZNTM&index=23
	Lesson 24	Use manipulatives to represent subtraction with decompositions of 1 hundred as 10 tens and 1 ten as 10 ones https://www.youtube.com/watch?v=EsqUeeQT2dw&list=PLvolZqLMhJmkb4rXaDraEy4I-TLolZNTM&index=24
	Lesson 25	Relate manipulative representations to a written method https://www.youtube.com/watch?v=tmfL5olqXQI&index=25&list=PLvolZqLMhJmkb4rXaDraEy4I-TLolZNTM
	Lesson 26	Use math drawings to represent subtraction with up to two decompositions and relate drawings to a written method https://www.youtube.com/watch?v=ZmjTmgGiPo0&list=PLvolZqLMhJmkb4rXaDraEy4I-TLolZNTM&index=26
	Lesson 27& 28	Subtract from 200 and from numbers with zeros in the tens place. https://www.youtube.com/watch?v=JHqUL9kRuco&index=27&list=PLvolZqLMhJmkb4rXaDraEy4I-TLolZNTM https://www.youtube.com/watch?v=XYLioeQHqmA&index=28&list=PLvolZqLMhJmkb4rXaDraEy4I-TLolZNTM
<p>Topic F: Student Explanations of Written Method</p>	Lesson 29	Use and explain the totals below method using words, math drawings, and numbers https://www.youtube.com/watch?v=ngJIR1da9c8&index=29&list=PLvolZqLMhJmkb4rXaDraEy4I-TLolZNTM
	Lesson 30	Lesson 30: Compare totals below to new groups below as written methods https://www.youtube.com/watch?v=t2mR2yi2Ams&index=30&list=PLvolZqLMhJmkb4rXaDraEy4I-TLolZNTM
	Lesson 31	Solve two-step word problems within 100 https://www.youtube.com/watch?v=3kpD56UsP7k&list=PLvolZqLMhJmkb4rXaDraEy4I-TLolZNTM&index=31
End-Module Assessment Task		

Modifications	
Special Education/ 504:	English Language Learners:
<ul style="list-style-type: none"> -Adhere to all modifications and health concerns stated in each IEP. -Give students a Menu, allowing students to pick assignments from different levels based on difficulty. -Accommodate Instructional Strategies: reading aloud text, graphic organizers, one-on-one instruction, class website (Google Classroom), handouts, definition list with visuals, extended time -Allow students to demonstrate understanding of a problem by drawing the picture of the answer and then explaining the reasoning orally and/or in writing , such as Read-Draw-Write -Provide breaks between tasks, use positive reinforcement, use proximity -Assure students have experiences that are on the Concrete- Pictorial- Abstract spectrum by using manipulatives -Common Core Approach to Differentiate Instruction: Students with Disabilities (pg 17-18) -Strategies for Students with 504 Plans 	<ul style="list-style-type: none"> - Use manipulatives to promote conceptual understanding and enhance vocabulary usage - Provide graphic representations, gestures, drawings, equations, realia, and pictures during all segments of instruction - During i-Ready lessons, click on “Español” to hear specific words in Spanish - Utilize graphic organizers which are concrete, pictorial ways of constructing knowledge and organizing information - Use sentence frames and questioning strategies so that students will explain their thinking/ process of how to solve word problems - Utilize program translations (if available) for L1/ L2 students - Reword questions in simpler language - Make use of the ELL Mathematical Language Routines (click here for additional information) -Scaffolding instruction for ELL Learners -Common Core Approach to Differentiate Instruction: Students with Disabilities (pg 16-17)
Gifted and Talented:	Students at Risk for Failure:
<ul style="list-style-type: none"> - Elevated contextual complexity - Inquiry based or open ended assignments and projects - More time to study concepts with greater depth - Promote the synthesis of concepts and making real world connections - Provide students with enrichment practice that are imbedded in the curriculum such as: <ul style="list-style-type: none"> ● Application / Conceptual Development ● Are you ready for more? - Common Core Approach to Differentiate Instruction: Students with Disabilities (pg. 20) - Provide opportunities for math competitions - Alternative instruction pathways available 	<ul style="list-style-type: none"> - Assure students have experiences that are on the Concrete- Pictorial- Abstract spectrum - Modify Instructional Strategies, reading aloud text, graphic organizers, one-on-one instruction, class website (Google Classroom), inclusion of more visuals and manipulatives, Field Trips, Google Expeditions, Peer Support, one on one instruction - Assure constant parental/ guardian contact throughout the year with successes/ challenges - Provide academic contracts to students and guardians - Create an interactive notebook with samples, key vocabulary words, student goals/ objectives. - Always plan to address students at risk in your learning tasks, instructions, and directions. Try to anticipate where the needs will be and then address them prior to lessons. -Common Core Approach to Differentiate Instruction: Students with Disabilities (pg 19)

21st Century Life and Career Skills:

Career Ready Practices describe the career-ready skills that all educators in all content areas should seek to develop in their students. They are practices that have been linked to increase college, career, and life success. Career Ready Practices should be taught and reinforced in all career exploration and preparation programs with increasingly higher levels of complexity and expectation as a student advances through a program of study.

<https://www.state.nj.us/education/cccs/2014/career/9.pdf>

- | | |
|--|--|
| <ul style="list-style-type: none">● CRP1. Act as a responsible and contributing citizen and employee.● CRP2. Apply appropriate academic and technical skills.● CRP3. Attend to personal health and financial well-being.● CRP4. Communicate clearly and effectively and with reason.● CRP5. Consider the environmental, social and economic impacts of decisions.● CRP6. Demonstrate creativity and innovation. | <ul style="list-style-type: none">● CRP7. Employ valid and reliable research strategies.● CRP8. Utilize critical thinking to make sense of problems and persevere in solving them.● CRP9. Model integrity, ethical leadership and effective management.● CRP10. Plan education and career paths aligned to personal goals.● CRP11. Use technology to enhance productivity.● CRP12. Work productively in teams while using cultural global competence. |
|--|--|

Students are given an opportunity to communicate with peers effectively, clearly, and with the use of technical language. They are encouraged to reason through experiences that promote critical thinking and emphasize the importance of perseverance. Students are exposed to various mediums of technology, such as digital learning, calculators, and educational websites.

Technology Standards:

All students will be prepared to meet the challenge of a dynamic global society in which they participate, contribute, achieve, and flourish through universal access to people, information, and ideas.

<https://www.state.nj.us/education/cccs/2014/tech/>

8.1 Educational Technology:

All students will use digital tools to access, manage, evaluate, and synthesize information in order to solve problems individually and collaborate and to create and communicate knowledge.

- A. **Technology Operations and Concepts:** Students demonstrate a sound understanding of technology concepts, systems and operations.
- B. **Creativity and Innovation:** Students demonstrate creative thinking, construct knowledge and develop innovative products and process using technology.
- C. **Communication and Collaboration:** Students use digital media and environments to communicate and work collaboratively, including at a distance, to support individual learning and contribute to the learning of others.
- D. **Digital Citizenship:** Students understand human, cultural, and societal issues related to technology and practice legal and ethical behavior.
- E. **Research and Information Fluency:** Students apply digital tools to gather, evaluate, and use of information.
- F. **Critical thinking, problem solving, and decision making:** Students use critical thinking skills to plan and conduct research, manage projects, solve problems, and make informed decisions using appropriate digital tools and resources.

8.2 Technology Education, Engineering, Design, and Computational Thinking - Programming:

All students will develop an understanding of the nature and impact of technology, engineering, technological design, computational thinking and the designed world as they relate to the individual, global society, and the environment.

- A. **The Nature of Technology: Creativity and Innovation-** Technology systems impact every aspect of the world in which we live.
- B. **Technology and Society:** Knowledge and understanding of human, cultural, and societal values are fundamental when designing technological systems and products in the global society.
- C. **Design:** The design process is a systematic approach to solving problems.
- D. **Abilities in a Technological World:** The designed world in a product of a design process that provides the means to convert resources into products and systems.
- E. **Computational Thinking: Programming-** Computational thinking builds and enhances problem solving, allowing students to move beyond using knowledge to creating knowledge.

Interdisciplinary Connections:	
English Language Arts:	
RF.2.4	Read with sufficient accuracy and fluency to support comprehension.
SL.2.1	Participate in collaborative conversations with diverse partners about <i>grade 2 topics and texts</i> with peers and adults in small and larger groups.
L.2.4	Determine or clarify the meaning of unknown and multiple-meaning words and phrases based on grade 2 reading and content, choosing flexibly from an array of strategies.

NJSL Standards Unpacked

2.OA.1

Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.¹

Second Grade students extend their work with addition and subtraction word problems in two major ways. First, they represent and solve word problems within 100, building upon their previous work to 20. In addition, they represent and solve one and two-step word problems of all three types (Result Unknown, Change Unknown, Start Unknown). Please see **Table 1** at end of document for examples of all problem types. One-step word problems use one operation. Two-step word problems use two operations which may include the same operation or opposite operations.

One Step Word Problem One Operation	Two-Step Word Problem Two Operations, Same	Two-Step Word Problem Two Operations, Opposite
<p>There are 15 stickers on the page. Brittany put some more stickers on the page. There are now 22 stickers on the page. How many stickers did Brittany put on the page?</p> <p style="text-align: center;"> $15 + \square = 22$ $22 - 15 = \square$ </p>	<p>There are 9 blue marbles and 6 red marbles in the bag. Maria put in 8 more marbles. How many marbles are in the bag now?</p> <p style="text-align: center;">$9 + 6 + 8 = \square$</p>	<p>There are 9 peas on the plate. Carlos ate 5 peas. Mother put 7 more peas on the plate. How many peas are on the plate now?</p> <p style="text-align: center;">$9 - 5 + 7 = \square$</p>

Two-Step Problems: Because Second Graders are still developing proficiency with the most difficult subtypes (shaded in white in Table 1 at end of the glossary): *Add To/Start Unknown*; *Take From/Start Unknown*; *Compare/Bigger Unknown*; and *Compare/Smaller Unknown*, two-step problems do **not** involve these sub-types (Common Core Standards Writing Team, May 2011). Furthermore, most two-step problems should focus on single-digit addends since the primary focus of the standard is the problem-type.

As second grade students solve one- and two-step problems they use manipulatives such as snap cubes, place value materials (groupable and pre-grouped), ten frames, etc.; create drawings of manipulatives to show their thinking; or use number lines to solve and describe their strategies. They then relate their drawings and materials to equations. By solving a variety of addition and subtraction word problems, second grade students determine the unknown in all positions (*Result unknown*, *Change unknown*, and *Start unknown*). Rather than a letter (“n”), boxes or pictures are used to represent the unknown number.

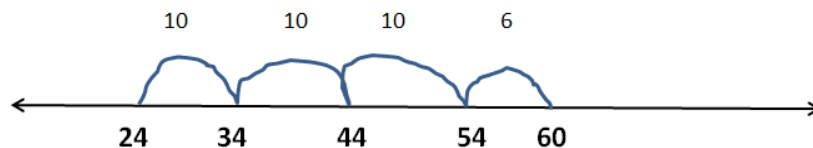
Problem Type: Add To		
<p><u>Result Unknown:</u> There are 29 students on the playground. Then 18 more students showed up. How many students are there now? $29 + 18 = \square$</p>	<p><u>Change Unknown:</u> There are 29 students on the playground. Some more students show up. There are now 47 students. How many students came? $29 + \star = 47$</p>	<p><u>Start Unknown:</u> There are some students on the playground. Then 18 more students came. There are now 47 students. How many students were on the playground at the beginning? $\square + 18 = 47$</p>

Second Graders use a range of methods, often mastering more complex strategies such as making tens and doubles and near doubles for problems involving addition and subtraction within 20. Moving beyond counting and counting-on, second grade students apply their understanding of place value to solve problems.

One-Step Example: Some students are in the cafeteria. 24 more students came in. Now there are 60 students in the cafeteria. How many were in the cafeteria to start with? Use drawings and equations to show your thinking.

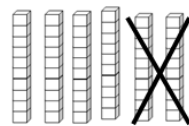
Student A: I read the equation and thought about how to write it with numbers. I thought, “What and 24 makes 60?” So, my equation for the problem is $\square + 24 = 60$. I used a number line to solve it.

I started with 24. Then I took jumps of 10 until I got close to 60. I landed on 54. Then, I took a jump of 6 to get to 60. So, $10 + 10 + 10 + 6 = 36$. So, there were 36 students in the cafeteria to start with.

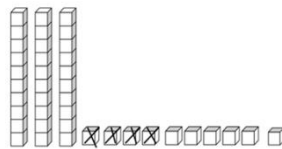


Student B: I read the equation and thought about how to write it with numbers. I thought, “There are 60 total. I know about the 24. So, what is $60 - 24$?” So, my equation for the problem is $60 - 24 = \square$. I used place value blocks to solve it.

I started with 60 and took 2 tens away.



I needed to take 4 more away. So, I broke up a ten into ten ones. Then, I took 4 away.

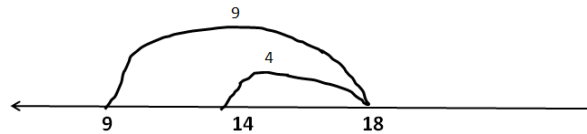


That left me with 36. So, 36 students were in the cafeteria at the beginning. $60 - 24 = 36$

Two-Step Example: There are 9 students in the cafeteria. 9 more students come in. After a few minutes, some students leave. There are now 14 students in the cafeteria. How many students left the cafeteria? Use drawings and equations to show your thinking.

Student A

I read the equation and thought about how to write it with numbers: $9 + 9 - \square = 14$. I used a number line to solve it. I started at 9 and took a jump of 9. I landed on 18. Then, I jumped back 4 to get to 14. So, overall, I took 4 jumps. 4 students left the cafeteria.



Student B

I read the equation and thought about how to write it with numbers: $9 + 9 - \square = 14$. I used doubles to solve it. I thought about double 9s. $9 + 9$ is 18. I knew that I only needed 14. So, I took 4 away, since 4 and 4 is eight. So, 4 students left the cafeteria.

2.NBT.5

Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction

There are various strategies that Second Grade students understand and use when adding and subtracting within 100 (such as those listed in the standard). The standard algorithm of carrying or borrowing is neither an expectation nor a focus in Second Grade. Students use multiple strategies for addition and subtraction in Grades K-3. By the end of Third Grade students use a range of algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction to fluently add and subtract within 1000. Students are expected to fluently add and subtract multi-digit whole numbers using the standard algorithm by the end of Grade 4.

Example: $67 + 25 = \underline{\quad}$

Place Value Strategy:

I broke both 67 and 25 into tens and ones. 6 tens plus 2 tens equals 8 tens. Then I added the ones. 7 ones plus 5 ones equals 12 ones. I then combined my tens and ones. 8 tens plus 12 ones equals 92.

Decomposing into Tens:

I decided to start with 67 and break 25 apart. I knew I needed 3 more to get to 70, so I broke off a 3 from the 25. I then added my 20 from the 22 left and got to 90. I had 2 left. 90 plus 2 is 92. So, $67 + 25 = 92$

Commutative Property:

I broke 67 and 25 into tens and ones so I had to add $60+7+20+5$. I added 60 and 20 first to get 80. Then I added 7 to get 87. Then I added 5 more. My answer is 92.

Example: $63 - 32 = \underline{\quad}$

Decomposing into Tens:

I broke apart both 63 and 32 into tens and ones. I know that 3 minus 2 is 1, so I have 1 left in the ones place. I know that 6 tens minus 3 tens is 3 tens, so I have a 3 in my tens place. My answer has a 1 in the ones place and 3 in the tens place, so my answer is 31. $63 - 32 = 31$

Think Addition:

I thought, '32 and what makes 63?'. I know that I needed 30, since 30 and 30 is 60. So, that got me to 62. I needed one more to get to 63. So, 30 and 1 is 31. $32 + 31 = 63$

2.NBT.6

Add up to four two-digit numbers using strategies based on place value and properties of operations

Second Grade students add a string of two-digit numbers (up to four numbers) by applying place value strategies and properties of operations.

Example: $43 + 34 + 57 + 24 = \underline{\quad}$

Student A

Associative Property

I saw the 43 and 57 and added them first. I know 3 plus 7 equals 10, so when I added them 100 was my answer. Then I added 34 and had 134. Then I added 24 and had 158.
 $43 + 57 + 34 + 24 = 158$

Student B

Place Value Strategies

I broke up all of the numbers into tens and ones. First I added the tens. $40 + 30 + 50 + 20 = 140$. Then I added the ones. $3 + 4 + 7 + 4 = 18$. That meant I had 1 ten and 8 ones. So, $140 + 10$ is 150. 150 and 8 more is 158. So, $43 + 34 + 57 + 24 = 158$

Student C

Place Value Strategies and Associative Property

I broke up all the numbers into tens and ones. First I added up the tens. $40 + 30 + 50 + 20$. I changed the order of the numbers to make adding easier. I know that 30 plus 20 equals 50 and 50 more equals 100. Then I added the 40 and got 140. Then I added up the ones. $3 + 4 + 7 + 4$. I changed the order of the numbers to make adding easier. I know that 3 plus 7 equals 10 and 4 plus 4 equals 8. 10 plus 8 equals 18. I then combined my tens and my ones. 140 plus 18 (1 ten and 8 ones) equals 158.

2.NBT.7

Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method. Understand that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds

Second graders extend the work from 2.NBT.5 to two 3-digit numbers. Students should have ample experiences using concrete materials and pictorial representations to support their work.

This standard also references composing and decomposing a ten. This work should include strategies such as making a 10, making a 100, breaking apart a 10, or creating an easier problem. The standard algorithm of carrying or borrowing is not an expectation in Second Grade. Students are not expected to add and subtract whole numbers using a standard algorithm until the end of Fourth Grade.

Example: $354 + 287 = \underline{\quad}$

Student A I started at 354 and jumped 200. I landed on 554. I then made 8 jumps of 10 and landed on 634. I then jumped 6 to land on 640. Then I jumped 1 more and landed on 641. $354 + 287 = 641$

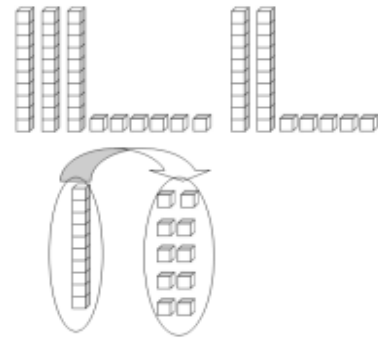
Student B I used place value blocks and a place value mat. I broke all of the numbers and placed them on the place value mat.
 I first added the ones. $4 + 7 = 11$.
 I then added the tens. $50 + 80 = 130$.
 I then added the hundreds. $300 + 200 = 500$.
 I then combined my answers. $500 + 130 = 630$. $630 + 11 = 641$.

Student C I used place value blocks. I made a pile of 354. I then added 287. That gave me 5 hundreds, 13 tens and 11 ones. I noticed that I could trade some pieces. I had 11 ones, and traded 10 ones for a ten. I then had 14 tens, so I traded 10 tens for a hundred. I ended up with 6 hundreds, 4 tens and 1 one. So, $354 + 287 = 641$

<p>2.NBT.8</p>	<p>Mentally add 10 or 100 to a given number 100–900, and mentally subtract 10 or 100 from a given number 100–900.</p>
<p>Second Grade students mentally add or subtract either 10 or 100 to any number between 100 and 900. As teachers provide ample experiences for students to work with pre-grouped objects and facilitate discussion, second graders realize that when one adds or subtracts 10 or 100 that only the tens place or the digit in the hundreds place changes by 1. As the teacher facilitates opportunities for patterns to emerge and be discussed, students notice the patterns and connect the digit change with the amount changed.</p> <p>Opportunities to solve problems in which students cross hundreds are also provided once students have become comfortable adding and subtracting within the same hundred</p> <p><u>Example: Within the same hundred</u></p> <p>What is 10 more than 218? What is 241 – 10?</p> <p><u>Example: Across hundreds</u></p> <p>293 + 10 = □ What is 10 less than 206?</p> <p>This standard focuses only on adding and subtracting 10 or 100. Multiples of 10 or multiples of 100 can be explored; however, the focus of this standard is to ensure that students are proficient with adding and subtracting 10 and 100 mentally</p>	
<p>2.NBT.9</p>	<p>Explain why addition and subtraction strategies work, using place value and the properties of operations.¹</p> <p>¹ Explanations may be supported by drawings or objects</p>
<p>Second graders explain why addition or subtraction strategies work as they apply their knowledge of place value and the properties of operations in their explanation. They may use drawings or objects to support their explanation.</p> <p>Once students have had an opportunity to solve a problem, the teacher provides time for students to discuss their strategies and why they did or didn't work.</p> <p><u>Example: There are 36 birds in the park. 25 more birds arrive. How many birds are there? Solve the problem and show your work.</u></p> <div data-bbox="126 1486 1401 1671" style="border: 1px solid black; padding: 5px;"> <p>Student A I broke 36 and 25 into tens and ones $30 + 6 + 20 + 5$. I can change the order of my numbers, since it doesn't change any amounts, so I added $30 + 20$ and got 50. Then I added 5 and 5 to make 10 and added it to the 50. So, 50 and 10 more is 60. I added the one that was left over and got on 6 to get 61. So there are 61 birds in the park.</p> </div>	

Student B

I used place value blocks and made a pile of 36 and a pile of 25. Altogether, I had 5 tens and 11 ones. 11 ones is the same as one ten and one left over. So, I really had 6 tens and 1 one. That makes 61.



Example: One of your classmates solved the problem $56 - 34 = \underline{\quad}$ by writing “I know that I need to add 2 to the number 4 to get 6. I also know that I need to add 20 to 30 to get 50. So, the answer is 22.” Is their strategy correct? Explain why or why not?

Student: I see what they did. Yes. I think the strategy is correct. They thought, ‘34 and what makes 56?’ So they thought about adding 2 to the 4 to get 6. Then, they had 36 and needed 56. So, they added 20 more. That means that they added 2 and 20 which is 22. I think that it’s right.

Example: One of your classmates solved the problem $25 + 35$ by adding $20 + 30 + 5 + 5$. Is their strategy correct? Explain why or why not?

Student: Well, $20 + 30$ is 50. And $5 + 5$ is 10. So, $50 + 10$ is 60. I got 60 too, but I did it a different way. I added 25 and 25 to make 50. Then I added 5 more and got 55. Then, I added 5 more and got 60. We both have 60. I think that it doesn’t matter if you add the 20 first or last. You still get the same amount.

M : Major Content

S: Supporting Content

A : Additional Content

Common addition and subtraction.¹

	RESULT UNKNOWN	CHANGE UNKNOWN	START UNKNOWN
ADD TO	Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? $2 + 3 = ?$	Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? $2 + ? = 5$	Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? $? + 3 = 5$
TAKE FROM	Five apples were on the table. I ate two apples. How many apples are on the table now? $5 - 2 = ?$	Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat? $5 - ? = 3$	Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before? $? - 2 = 3$
	TOTAL UNKNOWN	ADDEND UNKNOWN	BOTH ADDENDS UNKNOWN²
PUT TOGETHER / TAKE APART³	Three red apples and two green apples are on the table. How many apples are on the table? $3 + 2 = ?$	Five apples are on the table. Three are red and the rest are green. How many apples are green? $3 + ? = 5$, $5 - 3 = ?$	Grandma has five flowers. How many can she put in the red vase and how many in her blue vase? $5 = 0 + 5$, $5 = 1 + 4$, $5 = 4 + 1$, $5 = 2 + 3$, $5 = 3 + 2$
COMPARE	DIFFERENCE UNKNOWN	BIGGER UNKNOWN	SMALLER UNKNOWN
	("How many more?" version): Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy? ("How many fewer?" version): Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have than Julie? $2 + ? = 5$, $5 - 2 = ?$	(Version with "more"): Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have? (Version with "fewer"): Lucy has 3 fewer apples than Julie. Lucy has two apples. How many apples does Julie have? $2 + 3 = ?$, $3 + 2 = ?$	(Version with "more"): Julie has three more apples than Lucy. Julie has five apples. How many apples does Lucy have? (Version with "fewer"): Lucy has 3 fewer apples than Julie. Julie has five apples. How many apples does Lucy have? $5 - 3 = ?$, $? + 3 = 5$

¹ Adapted from Box 2-4 of Mathematics Learning in Early Childhood, National Research Council (2009, pp. 32, 33).

² These take apart situations can be used to show all the decompositions of a given number. The associated equations, which have the total on the left of the equal sign, help children understand that the $-$ sign does not always mean, makes or results in but always does mean is the same number as.

³ Either addend can be unknown, so there are three variations of these problem situations. Both addends Unknown is a productive extension of the basic situation, especially for small numbers less than or equal to 10.

⁴ For the Bigger Unknown or Smaller Unknown situations, one version directs the correct operation (the version using more for the bigger unknown and using less for the smaller unknown). The other versions are more difficult.

<http://www.corestandards.org/Math/Content/mathematics-glossary/Table-1/>

Module 4 Assessment Framework			
Assessment	NJSLS	Estimated Time	Format
Diagnostic Assessment 2 (iReady)		1-2 blocks	Individual
Optional Mid- Module Assessment (Interview Style)	2.OA.1 2.NBT.5 2.NBT.7 2.NBT.8 2.NBT.9	1 Block	Individual
Optional End-of- Module Assessment (Interview Style)	2.OA.1 2.NBT.5 2.NBT.6 2.NBT.7 2.NBT.8 2.NBT.9	1 Block	Individual or Small Group with Teacher
Grade 2 Interim Assessment 3	2.NBT.1-5	1 Block	Individual

Module 4 Performance Assessment/ PBL Framework			
Assessment	NJSLS	Estimated Time	Format
Module 4 Performance Task 1 <i>Counting Mice</i>	2.OA.1 2.NBT.5	Up to 30 minutes	Individual or Small Group
Extended Constructed Response (ECR)* (click here for access)	Dependent on unit of study & month of administration	Up to 30 Minutes	Individual

Use the following links to access ECR protocol and district assessment scoring documents:

- [Assessment and Data in Mathematics Bulletin](#)
- [ECR Protocol](#)

Second Grade Ideal Math Block

Fluency: Whole Group
Sprints, Counting, Whiteboard Exchange

Application Problem: Whole Group
Provides HANDS-ON work to allow children to ACT OUT or ENGAGE ACTIVELY with the new MATH IDEA
Technology Integration: <https://embarc.online/>
*Website provides Goggle slides, additional activities, and student videos per lesson

Concept Development: Individual/partner/whole
Instruction & Strategic Problem Set Questions
Technology Integration: <https://embarc.online/>
Website provides Goggle slides, additional activities, and student videos. per lesson

Student Debrief: Whole Group
Exit Ticket: Independent

CENTERS/STATIONS:
Pairs / Small Group/ Individual
DIFFERENTIATED activities designed to RETEACH, REMEDIATE, ENRICH student's understanding of concepts.

M: Meet with the teacher https://teacher-toolbox.com/	A: Application/ Problem Solving	T: Technology Resources I-ready Zearn	H: Hands on Activities
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50-60 min.

20-30 min.

Lesson Structure:

Fluency:

- Sprints
- Whiteboard Exchange

Technology Integration:

Splat Sequences

[Which one doesn't belong?](#)

[Would you rather?](#)

Esti- Mysteries

Anchor Task:

- Engage students in using the RDW Process
- Sequence problems from simple to complex and adjust based on students' responses
- Facilitate share and critique of various explanations, representations, and/or examples.

Guided Practice/ Independent Practice : (largest chunk of time)

Instruction:

- Maintain overall alignment with the objectives and suggested pacing and structure.
- Use of tools, precise mathematical language, and/or models
- Balance teacher talk with opportunities for peer share and/or collaboration
- Generate next steps by watching and listening for understanding

Problem Set: (Individual, partner, or group)

- Allow for independent practice and productive struggle
- Assign problems strategically to differentiate practice as needed
- Create and assign remedial sequences as needed

Technology Integration:

Think Central:

- Pre-Test
- Chapter Review
- Test Prep
- Performance Tasks

<https://embarc.online/>

[Virtual Manipulatives](#) for lessons

<http://nlvm.usu.edu/en/nav/vlibrary.html>

For videos that students can watch and interact with independently click [here](#)

Student Debrief:

- Elicit students thinking, prompt reflection, and promote metacognition through student centered discussion
- Culminate with students' verbal articulation of their learning for the day
- Close with completion of the daily Exit Ticket (opportunity for informal assessment that guides effective preparation of subsequent lessons) as needed.

Centers:

- I-Ready: <https://login.i-ready.com/> *i-Ready* makes the promise of differentiated instruction a practical reality for teachers and students. It was designed to get students excited about learning and to support teachers in the challenge of meeting the needs of all learners. Through the power of one intuitive system whose pieces were built from the ground up to work together, teachers have the tools they need to ensure students are on the road to proficiency.
- Zearn: <https://www.zearn.org/> Zearn Math is a K-5 math curriculum based on Eureka Math with top-rated materials for teacher-led and digital instruction.
- Teacher Toolbox; <https://teacher-toolbox.com/> A digital collection of K-8 resources to help you differentiate instruction to students performing on, below, and above grade level.

NJSLA Assessment Evidence/Clarification Statements			
NJSLS	Evidence Statement	Clarification	Math Practices
2.NBT.5	Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.	<ul style="list-style-type: none"> • Tasks do not have a context. • Sums and differences beyond 20 but within 100 should be emphasized in 75% of the tasks. • Only the answer is required (strategies, representations, etc. are not assessed here).. 	MP 7,8
2.NBT.6	Add up to four two-digit numbers using strategies based on place value and properties of operation	<ul style="list-style-type: none"> • Tasks do not have a context. • Only the answer is required (strategies, representations, etc. are not assessed here) 	MP 7,8
2.NBT.7	Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method. Understand that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundred	<ul style="list-style-type: none"> • Emphasis is on adding and subtracting hundreds. • Tasks do not have a context. 	MP 7,8

Number Talks

What does Number Talks look like?

- Students are near each other so they can communicate with each other (central meeting place)
- Students are mentally solving problems
- Students are given thinking time
- Thumbs up show when they are ready
- Teacher is recording students' thinking

Communication

- Having to talk out loud about a problem helps students clarify their own thinking
- Allow students to listen to other's strategies and value other's thinking
- Gives the teacher the opportunity to hear student's thinking

Mental Math

- When you are solving a problem mentally you must rely on what you know and understand about the numbers instead of memorized procedures
- You must be efficient when computing mentally because you can hold a lot of quantities in your head

Thumbs Up

- This is just a signal to let you know that you have given your students enough time to think about the problem
- It will give you a picture of who is able to compute mentally and who is struggling
- It isn't as distracting as a waving hand

Teacher as Recorder

- Allows you to record students' thinking in the correct notation
- Provides a visual to look at and refer back to
- Allows you to keep a record of the problems posed and which students offered specific strategies

Purposeful Problems

- Start with small numbers so the students can learn to focus on the strategies instead of getting lost in the numbers
- Use a number string (a string of problems that are related to and scaffold each other)

Starting Number Talks in your Classroom

- Start with specific problems in mind
- Be prepared to offer a strategy from a previous student
- It is ok to put a student's strategy on the backburner
- Limit your number talks to about 15 minutes
- Ask a question, don't tell!

The teacher asks questions:

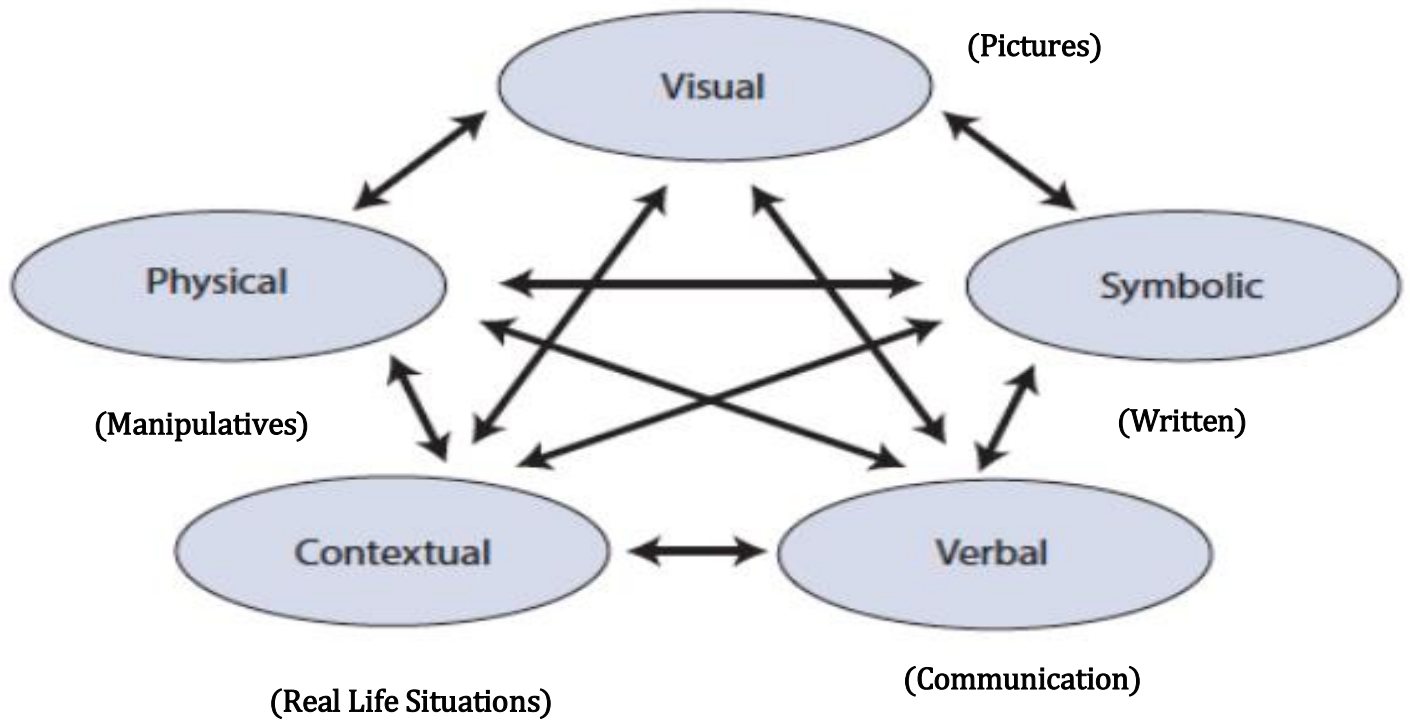
- Who would like to share their thinking?
- Who did it another way?
- How many people solved it the same way as Billy?
- Does anyone have any questions for Billy?
- Billy, can you tell us where you got that 5?
- How did you figure that out?

Student Name: _____ Task: _____ School: _____ Teacher: _____

Date: _____

"I CAN...."	STUDENT FRIENDLY RUBRIC				SCORE
	...a start 1	...getting there 2	...that's it 3	WOW! 4	
Understand	I need help.	I need some help.	I do not need help.	I can help a classmate.	
Solve	I am unable to use a strategy.	I can start to use a strategy.	I can solve it more than one way.	I can use more than one strategy and talk about how they get to the same answer.	
Say or Write	I am unable to say or write.	I can write or say some of what I did.	I can write and talk about what I did. I can write or talk about why I did it.	I can write and say what I did and why I did it.	
Draw or Show	I am not able to draw or show my thinking.	I can draw, but not show my thinking; or I can show but not draw my thinking;	I can draw and show my thinking	I can draw, show and talk about my thinking.	

Use and Connection of Mathematical Representations



The Lesh Translation Model

Each oval in the model corresponds to one way to represent a mathematical idea.

Visual: When children draw pictures, the teacher can learn more about what they understand about a particular mathematical idea and can use the different pictures that children create to provoke a discussion about mathematical ideas. Constructing their own pictures can be a powerful learning experience for children because they must consider several aspects of mathematical ideas that are often assumed when pictures are pre-drawn for students.

Physical: The manipulatives representation refers to the unifix cubes, base-ten blocks, fraction circles, and the like, that a child might use to solve a problem. Because children can physically manipulate these objects, when used appropriately, they provide opportunities to compare relative sizes of objects, to identify patterns, as well as to put together representations of numbers in multiple ways.

Verbal: Traditionally, teachers often used the spoken language of mathematics but rarely gave students opportunities to grapple with it. Yet, when students do have opportunities to express their mathematical reasoning aloud, they may be able to make explicit some knowledge that was previously implicit for them.

Symbolic: Written symbols refer to both the mathematical symbols and the written words that are associated with them. For students, written symbols tend to be more abstract than the other representations. I tend to introduce symbols after students have had opportunities to make connections among the other representations, so that the students have multiple ways to connect the symbols to mathematical ideas, thus increasing the likelihood that the symbols will be comprehensible to students.

Contextual: A relevant situation can be any context that involves appropriate mathematical ideas and holds interest for children; it is often, but not necessarily, connected to a real-life situation.

The Lesh Translation Model: Importance of Connections

As important as the ovals are in this model, another feature of the model is even more important than the representations themselves: The arrows! The arrows are important because they represent the connections students make between the representations. When students make these connections, they may be better able to access information about a mathematical idea, because they have multiple ways to represent it and, thus, many points of access.

Individuals enhance or modify their knowledge by building on what they already know, so the greater the number of representations with which students have opportunities to engage, the more likely the teacher is to tap into a student's prior knowledge. This "tapping in" can then be used to connect students' experiences to those representations that are more abstract in nature (such as written symbols). Not all students have the same set of prior experiences and knowledge. Teachers can introduce multiple representations in a meaningful way so that students' opportunities to grapple with mathematical ideas are greater than if their teachers used only one or two representations.

Concrete Pictorial Abstract (CPA) Instructional Approach

The CPA approach suggests that there are three steps necessary for pupils to develop understanding of a mathematical concept.

Concrete: “Doing Stage”: Physical manipulation of objects to solve math problems.

Pictorial: “Seeing Stage”: Use of imaged to represent objects when solving math problems.

Abstract: “Symbolic Stage”: Use of only numbers and symbols to solve math problems.

CPA is a gradual systematic approach. Each stage builds on to the previous stage. Reinforcement of concepts are achieved by going back and forth between these representations and making connections between stages. Students will benefit from seeing parallel samples of each stage and how they transition from one to another.

Read, Draw, Write Process

READ the problem. Read it over and over.... And then read it again.

DRAW a picture that represents the information given. During this step students ask themselves: Can I draw something from this information? What can I draw? What is the best model to show the information? What conclusions can I make from the drawing?

WRITE your conclusions based on the drawings. This can be in the form of a number sentence, an equation, or a statement.

Students are able to draw a model of what they are reading to help them understand the problem. Drawing a model helps students see which operation or operations are needed, what patterns might arise, and which models work and do not work. Students must dive deeper into the problem by drawing models and determining which models are appropriate for the situation.

While students are employing the RDW process they are using several Standards for Mathematical Practice and in some cases, all of them.

Mathematical Discourse and Strategic Questioning Discourse involves asking strategic questions that elicit from students their understanding of the context and actions taking place in a problem, how a problem is solved and why a particular method was chosen. Students learn to critique their own and others' ideas and seek out efficient mathematical solutions.

While classroom discussions are nothing new, the theory behind classroom discourse stems from constructivist views of learning where knowledge is created internally through interaction with the environment. It also fits in with socio-cultural views on learning where students working together are able to reach new understandings that could not be achieved if they were working alone.

Underlying the use of discourse in the mathematics classroom is the idea that mathematics is primarily about reasoning not memorization. Mathematics is not about remembering and applying a set of procedures but about developing understanding and explaining the processes used to arrive at solutions.

Teacher Questioning:

Asking better questions can open new doors for students, promoting mathematical thinking and classroom discourse. Can the questions you're asking in the mathematics classroom be answered with a simple “yes” or “no,” or do they invite students to deepen their understanding?



The most
important thing
is to NEVER
stop
questioning

Albert Einstein

To help you encourage deeper discussions, here are 100 questions to incorporate into your instruction by Gladis Kersaint, mathematics expert and advisor for Ready Mathematics.

Dr.

100 questions that promote

Mathematical Discourse

Help students **work together** to make sense of mathematics

- 1 What **strategy** did you use?
- 2 Do you **agree**?
- 3 Do you **disagree**?
- 4 Would you **ask the rest of the class** that question?
- 5 Could you **share your method** with the class?
- 6 What part of what he said **do you understand**?
- 7 Would someone like to **share** ___?
- 8 Can you **convince the rest of us** that your answer makes sense?
- 9 **What do others think** about what [student] said?
- 10 Can someone **retell or restate** [student]'s explanation?
- 11 Did you **work together**? In what way?
- 12 Would anyone like to **add to what was said**?
- 13 Have you **discussed** this with your group? With others?
- 14 Did anyone get a **different answer**?
- 15 **Where** would you go for **help**?
- 16 **Did everybody get a fair chance** to talk, use the manipulatives, or be the recorder?
- 17 How could you help another student **without telling them the answer**?
- 18 **How would you explain** ___ to someone who missed class today?

Help students **rely more on themselves** to determine whether something is mathematically correct

- 19 Is this a **reasonable answer**?
- 20 Does that make **sense**?
- 21 **Why** do you think that? Why is that true?
- 22 Can you **draw a picture or make a model** to show that?
- 23 **How** did you reach that conclusion?
- 24 Does anyone want to **revise** his or her answer?
- 25 **How were you sure** your answer was right?

Ready

Help students learn to reason mathematically

- 26 How did you **begin** to think about this problem?
- 27 What is **another way** you could solve this problem?
- 28 How could you **prove** _____?
- 29 Can you **explain how your answer is different from or the same as** [student]'s answer?
- 30 Let's **break the problem into parts**. What would the parts be?
- 31 Can you **explain this part more specifically**?
- 32 Does that **always work**?
- 33 Can you think of a case where that **wouldn't work**?
- 34 How did you **organize** your information? Your thinking?

Help students with problem comprehension

- 39 What is this problem about? What can you **tell me about it**?
- 40 Do you need to **define or set limits** for the problem?
- 41 How would you **interpret** that?
- 42 Could you **reword that in simpler terms**?
- 43 Is there something that can be **eliminated** or that is **missing**?
- 44 Could you **explain** what the problem is asking?
- 45 What **assumptions** do you have to make?
- 46 What do you **know** about this part?
- 47 Which words were **most important**? Why?

Help students evaluate their own processes and engage in productive peer interaction

- 35 What do you need to do **next**?
- 36 What have you **accomplished**?
- 37 What are your **strengths and weaknesses**?
- 38 Was your **group participation appropriate and helpful**?



Help students learn to **conjecture, invent, and solve problems**

- 48 What would happen if ___?
- 49 Do you see a **pattern**?
- 50 What are some **possibilities** here?
- 51 Where could you find the **information** you need?
- 52 How would you **check your steps** or your answer?
- 53 What **did not work**?
- 54 How is your solution method the **same as or different from** [student]'s method?
- 55 Other than retracing your steps, **how can you determine** if your answers are appropriate?
- 56 How did you **organize** the information? Do you have a **record**?
- 57 How could you solve this using **tables, lists, pictures, diagrams**, etc.?
- 58 What have you tried? What **steps** did you take?
- 59 How would it look if you used this **model** or these **materials**?
- 60 How would you draw a **diagram or make a sketch** to solve the problem?
- 61 Is there **another possible answer**? If so, explain.
- 62 Is there **another way to solve** the problem?
- 63 Is there **another model** you could use to solve the problem?
- 64 Is there anything you've **overlooked**?
- 65 **How did you think** about the problem?
- 66 What was your **estimate or prediction**?
- 67 How **confident** are you in your answer?
- 68 **What else** would you like to know?
- 69 What do you think comes **next**?
- 70 Is the solution **reasonable**, considering the context?
- 71 Did you have a **system**? Explain it.
- 72 Did you have a **strategy**? Explain it.
- 73 Did you have a **design**? Explain it.



Help students learn to **connect mathematics, its ideas, and its application**

- 74 What is the **relationship** between ___ and ___?
- 75 Have we ever solved a problem **like this before**?
- 76 What uses of mathematics did you find in the **newspaper** last night?
- 77 What is the **same**?
- 78 What is **different**?
- 79 Did you use skills or build on concepts that were **not necessarily mathematical**?
- 80 Which **skills or concepts** did you use?
- 81 What **ideas** have we explored before that were useful in solving this problem?
- 82 Is there a **pattern**?
- 83 **Where else** would this strategy be useful?
- 84 How does this **relate** to ___?
- 85 Is there a **general rule**?
- 86 Is there a **real-life situation** where this could be used?
- 87 How would your method work with **other problems**?
- 88 What other problem does this seem to **lead to**?

Help students **persevere**

- 89 Have you tried making a **guess**?
- 90 **What else** have you tried?
- 91 Would **another method** work as well or better?
- 92 Is there **another way** to draw, explain, or say that?
- 93 Give me another **related problem**. Is there an easier problem?
- 94 How would you **explain** what you know right now?
- 95 What was **one thing you learned** (or two, or more)?
- 96 Did you **notice any patterns**? If so, describe them.
- 97 What **mathematics topics** were used in this investigation?
- 98 What were the **mathematical ideas** in this problem?
- 99 What is mathematically **different about these two situations**?
- 100 What are the **variables** in this problem? What stays **constant**?

Help students **focus on the mathematics from activities**

Conceptual Understanding

Students demonstrate conceptual understanding in mathematics when they provide evidence that they can:

- recognize, label, and generate examples of concepts;
- use and interrelate models, diagrams, manipulatives, and varied representations of concepts;
- identify and apply principles; know and apply facts and definitions;
- compare, contrast, and integrate related concepts and principles; and
- recognize, interpret, and apply the signs, symbols, and terms used to represent concepts.

Conceptual understanding reflects a student's ability to reason in settings involving the careful application of concept definitions, relations, or representations of either.

Procedural Fluency

Procedural fluency is the ability to:

- apply procedures accurately, efficiently, and flexibly;
- to transfer procedures to different problems and contexts;
- to build or modify procedures from other procedures; and
- to recognize when one strategy or procedure is more appropriate to apply than another.

Procedural fluency is more than memorizing facts or procedures, and it is more than understanding and being able to use one procedure for a given situation. Procedural fluency builds on a foundation of conceptual understanding, strategic reasoning, and problem solving (NGA Center & CCSSO, 2010; NCTM, 2000, 2014). Research suggests that once students have memorized and practiced procedures that they do not understand, they have less motivation to understand their meaning or the reasoning behind them (Hiebert, 1999). Therefore, the development of students' conceptual understanding of procedures should precede and coincide with instruction on procedures.

Math Fact Fluency: Automaticity

Students who possess math fact fluency can recall math facts with automaticity. Automaticity is the ability to do things without occupying the mind with the low-level details required, allowing it to become an automatic response pattern or habit. It is usually the result of learning, repetition, and practice.

K-2 Math Fact Fluency Expectation

K.OA.5 Add and Subtract within 5.

1.OA.6 Add and Subtract within 10.

2.OA.2 Add and Subtract within 20.

Math Fact Fluency: Fluent Use of Mathematical Strategies

First and second grade students are expected to solve addition and subtraction facts using a variety of strategies fluently.

1.OA.6 Add and subtract within 20, demonstrating fluency for addition and subtraction within 10.

Use strategies such as:

- counting on; making ten (e.g., $8 + 6 = 8 + 2 + 4 = 10 + 4 = 14$);
- decomposing a number leading to a ten (e.g., $13 - 4 = 13 - 3 - 1 = 10 - 1 = 9$);
- using the relationship between addition and subtraction; and
- creating equivalent but easier or known sums.

2.NBT.7 Add and subtract within 1000, using concrete models or drawings and strategies based on:

- place value,
- properties of operations, and/or
- the relationship between addition and subtraction;

Evidence of Student Thinking

Effective classroom instruction and more importantly, improving student performance, can be accomplished when educators know how to elicit evidence of students' understanding on a daily basis. Informal and formal methods of collecting evidence of student understanding enable educators to make positive instructional changes. An educators' ability to understand the processes that students use helps them to adapt instruction allowing for student exposure to a multitude of instructional approaches, resulting in higher achievement. By highlighting student thinking and misconceptions, and eliciting information from more students, all teachers can collect more representative evidence and can therefore better plan instruction based on the current understanding of the entire class.

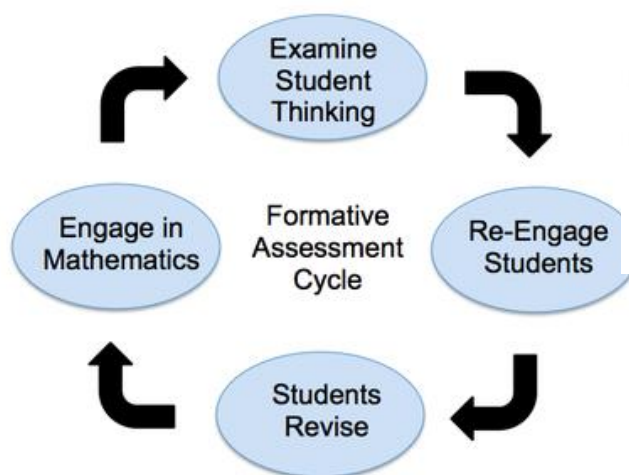
Mathematical Proficiency

To be mathematically proficient, a student must have:

- Conceptual understanding: comprehension of mathematical concepts, operations, and relations;
- Procedural fluency: skill in carrying out procedures flexibly, accurately, efficiently, and appropriately;
- Strategic competence: ability to formulate, represent, and solve mathematical problems;
- Adaptive reasoning: capacity for logical thought, reflection, explanation, and justification;
- Productive disposition: habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy.

Evidence should:

- Provide a window in student thinking;
- Help teachers to determine the extent to which students are reaching the math learning goals; and
- Be used to make instructional decisions during the lesson and to prepare for subsequent lessons.



Formative assessment is an essentially interactive process, in which the teacher can find out whether what has been taught has been learned, and if not, to do something about it. Day-to-day formative assessment is one of the most powerful ways of improving learning in the mathematics classroom.

(William 2007, pp. 1054; 1091)

Connections to the Mathematical Practices

Student Friendly Connections to the Mathematical Practices

1. I can solve problems without giving up.
2. I can think about numbers in many ways.
3. I can explain my thinking and try to understand others.
4. I can show my work in many ways.
5. I can use math tools and tell why I choose them.
6. I can work carefully and check my work.
7. I can use what I know to solve new problems.
8. I can discover and use short cuts.

The **Standards for Mathematical Practice** describe varieties of expertise that mathematics educators at all levels should seek to develop in their students.

Make sense of problems and persevere in solving them

1

In Kindergarten, students learn that doing math involves solving problems and discussing how they solved them. Students will begin to explain the meaning of a problem, and look for ways to solve it. Kindergarteners will learn how to use objects and pictures to help them understand and solve problems. They will begin to check their thinking when the teacher asks them how they got their answer, and if the answer makes sense. When working in small groups or with a partner they will listen to the strategies of the group and will try different approaches.

Reason abstractly and quantitatively

2

Mathematically proficient students in Kindergarten make sense of quantities and the relationships while solving tasks. This involves two processes- decontextualizing and contextualizing. In Kindergarten, students represent situations by decontextualizing tasks into numbers and symbols. For example, in the task, “There are 7 children on the playground and some children go line up. If there are 4 children still playing, how many children lined up?” Kindergarten students are expected to translate that situation into the equation: $7-4 = \underline{\quad}$, and then solve the task. Students also contextualize situations during the problem solving process. For example, while solving the task above, students refer to the context of the task to determine that they need to subtract 4 since the number of children on the playground is the total number of students except for the 4 that are still playing. Abstract reasoning also occurs when students measure and compare the lengths of objects.

3	<p>Construct viable arguments and critique the reasoning of others</p> <p>Mathematically proficient students in Kindergarten accurately use mathematical terms to construct arguments and engage in discussions about problem solving strategies. For example, while solving the task, “There are 8 books on the shelf. If you take some books off the shelf and there are now 3 left, how many books did you take off the shelf?” students will solve the task, and then be able to construct an accurate argument about why they subtracted 3 from 8 rather than adding 8 and 3. Further, Kindergarten students are expected to examine a variety of problem solving strategies and begin to recognize the reasonableness of them, as well as similarities and differences among them.</p>
4	<p>Model with mathematics</p> <p>Mathematically proficient students in Kindergarten model real-life mathematical situations with a number sentence or an equation, and check to make sure that their equation accurately matches the problem context. Kindergarten students rely on concrete manipulatives and pictorial representations while solving tasks, but the expectation is that they will also write an equation to model problem situations. For example, while solving the task “there are 7 bananas on the counter. If you eat 3 bananas, how many are left?” Kindergarten students are expected to write the equation $7-3 = 4$. Likewise, Kindergarten students are expected to create an appropriate problem situation from an equation. For example, students are expected to orally tell a story problem for the equation $4+5 = 9$.</p>
5	<p>Use appropriate tools strategically</p> <p>Mathematically proficient students in Kindergarten have access to and use tools appropriately. These tools may include counters, place value (base ten) blocks, hundreds number boards, number lines, and concrete geometric shapes (e.g., pattern blocks, 3-d solids). Students should also have experiences with educational technologies, such as calculators, virtual manipulatives, and mathematical games that support conceptual understanding. During classroom instruction, students should have access to various mathematical tools as well as paper, and determine which tools are the most appropriate to use. For example, while solving the task “There are 4 dogs in the park. If 3 more dogs show up, how many dogs are they?” Kindergarten students are expected to explain why they used specific mathematical tools.”</p>
6	<p>Attend to precision</p> <p>Mathematically proficient students in Kindergarten are precise in their communication, calculations, and measurements. In all mathematical tasks, students in Kindergarten describe their actions and strategies clearly, using grade-level appropriate vocabulary accurately as well as giving precise explanations and reasoning regarding their process of finding solutions. For example, while measuring objects iteratively (repetitively), students check to make sure that there are no gaps or overlaps. During tasks involving number sense, students check their work to ensure the accuracy and reasonableness of solutions.</p>

7	<p>Look for and make use of structure</p> <p>Mathematically proficient students in Kindergarten carefully look for patterns and structures in the number system and other areas of mathematics. While solving addition problems, students begin to recognize the commutative property, in that $1+4 = 5$, and $4+1 = 5$. While decomposing teen numbers, students realize that every number between 11 and 19, can be decomposed into 10 and some leftovers, such as $12 = 10+2$, $13 = 10+3$, etc. Further, Kindergarten students make use of structures of mathematical tasks when they begin to work with subtraction as missing addend problems, such as $5 - 1 = \underline{\quad}$ can be written as $1 + \underline{\quad} = 5$ and can be thought of as how much more do I need to add to 1 to get to 5?</p>
8	<p>Look for and express regularity in repeated reasoning</p> <p>Mathematically proficient students in Kindergarten begin to look for regularity in problem structures when solving mathematical tasks. Likewise, students begin composing and decomposing numbers in different ways. For example, in the task “There are 8 crayons in the box. Some are red and some are blue. How many of each could there be?” Kindergarten students are expected to realize that the 8 crayons could include 4 of each color ($4+4 = 8$), 5 of one color and 3 of another ($5+3 = 8$), etc. For each solution, students repeated engage in the process of finding two numbers that can be joined to equal 8.</p>

Effective Mathematics Teaching Practices

Establish mathematics goals to focus learning. Effective teaching of mathematics establishes clear goals for the mathematics that students are learning, situates goals within learning progressions, and uses the goals to guide instructional decisions.

Implement tasks that promote reasoning and problem solving. Effective teaching of mathematics engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies.

Use and connect mathematical representations. Effective teaching of mathematics engages students in making connections among mathematical representations to deepen understanding of mathematics concepts and procedures and as tools for problem solving.

Facilitate meaningful mathematical discourse. Effective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments.

Pose purposeful questions. Effective teaching of mathematics uses purposeful questions to assess and advance students' reasoning and sense making about important mathematical ideas and relationships.

Build procedural fluency from conceptual understanding. Effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding so that students, over time, become skillful in using procedures flexibly as they solve contextual and mathematical problems.

Support productive struggle in learning mathematics. Effective teaching of mathematics consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships.

Elicit and use evidence of student thinking. Effective teaching of mathematics uses evidence of student thinking to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning.

5 Practices for Orchestrating Productive Mathematics Discussions

Practice	Description/ Questions
1. Anticipating	<p>What strategies are students likely to use to approach or solve a challenging high-level mathematical task?</p> <p>How do you respond to the work that students are likely to produce?</p> <p>Which strategies from student work will be most useful in addressing the mathematical goals?</p>
2. Monitoring	<p>Paying attention to what and how students are thinking during the lesson.</p> <p>Students working in pairs or groups</p> <p>Listening to and making note of what students are discussing and the strategies they are using</p> <p>Asking students questions that will help them stay on track or help them think more deeply about the task. (Promote productive struggle)</p>
3. Selecting	<p>This is the process of deciding the <i>what</i> and the <i>who</i> to focus on during the discussion.</p>
4. Sequencing	<p>What order will the solutions be shared with the class?</p>
5. Connecting	<p>Asking the questions that will make the mathematics explicit and understandable.</p> <p>Focus must be on mathematical meaning and relationships; making links between mathematical ideas and representations.</p>

MATH CENTERS/ WORKSTATIONS

Math workstations allow students to engage in authentic and meaningful hands-on learning. They often last for several weeks, giving students time to reinforce or extend their prior instruction. Before students have an opportunity to use the materials in a station, introduce them to the whole class, several times. Once they have an understanding of the concept, the materials are then added to the work stations.

Station Organization and Management Sample

Teacher A has 12 containers labeled 1 to 12. The numbers correspond to the numbers on the rotation chart. She pairs students who can work well together, who have similar skills, and who need more practice on the same concepts or skills. Each day during math work stations, students use the center chart to see which box they will be using and who their partner will be. Everything they need for their station will be in their box. **Each station is differentiated.** If students need more practice and experience working on numbers 0 to 10, those will be the only numbers in their box. If they are ready to move on into the teens, then she will place higher number activities into the box for them to work with.



In the beginning there is a lot of prepping involved in gathering, creating, and organizing the work stations. However, once all of the initial work is complete, the stations are easy to manage. Many of her stations stay in rotation for three or four weeks to give students ample opportunity to master the skills and concepts.

Read *Math Work Stations* by Debbie Diller.

In her book, she leads you step-by-step through the process of implementing work stations.

MATH WORKSTATION INFORMATION CARD

Math Workstation: _____

Time: _____

NJSLS:

Objective(s): By the end of this task, I will be able to:

- _____
- _____
- _____

Task(s):

- _____
- _____
- _____
- _____

Exit Ticket:

- _____
- _____
- _____

MATH WORKSTATION SCHEDULE

Week of: _____

DAY	Technology Lab	Problem Solving Lab	Fluency Lab	Math Journal	Small Group Instruction
Mon.	Group ____	Group ____	Group ____	Group ____	BASED ON CURRENT OB- SERVATIONAL DA- TA
Tues.	Group ____	Group ____	Group ____	Group ____	
Wed.	Group ____	Group ____	Group ____	Group ____	
Thurs.	Group ____	Group ____	Group ____	Group ____	
Fri.	Group ____	Group ____	Group ____	Group ____	

INSTRUCTIONAL GROUPING

	GROUP A		GROUP B
1		1	
2		2	
3		3	
4		4	
5		5	
6		6	
	GROUP C		GROUP D
1		1	
2		2	
3		3	
4		4	
5		5	

Second Grade PLD Rubric

Got It		Not There Yet		
Evidence shows that the student essentially has the target concept or big math idea.		Student shows evidence of a major misunderstanding, incorrect concepts or procedure, or a failure to engage in the task.		
PLD Level 5: 100% Distinguished command	PLD Level 4: 89% Strong Command	PLD Level 3: 79% Moderate Command	PLD Level 2: 69% Partial Command	PLD Level 1: 59% Little Command
<p>Student work shows distinct levels of understanding of the mathematics.</p> <p>Student constructs and communicates a complete response based on explanations/reasoning using the:</p> <ul style="list-style-type: none"> • Tools: <ul style="list-style-type: none"> ○ Manipulatives ○ Five Frame ○ Ten Frame ○ Number Line ○ Part-Part-Whole Model • Strategies: <ul style="list-style-type: none"> ○ Drawings ○ Counting All ○ Count On/Back ○ Skip Counting ○ Making Ten ○ Decomposing Number • Precise use of math vocabulary <p>Response includes an efficient and logical progression of mathematical reasoning and understanding.</p>	<p>Student work shows strong levels of understanding of the mathematics.</p> <p>Student constructs and communicates a complete response based on explanations/reasoning using the:</p> <ul style="list-style-type: none"> • Tools: <ul style="list-style-type: none"> ○ Manipulatives ○ Five Frame ○ Ten Frame ○ Number Line ○ Part-Part-Whole Model • Strategies: <ul style="list-style-type: none"> ○ Drawings ○ Counting All ○ Count On/Back ○ Skip Counting ○ Making Ten ○ Decomposing Number • Precise use of math vocabulary <p>Response includes a logical progression of mathematical reasoning and understanding.</p>	<p>Student work shows moderate levels of understanding of the mathematics.</p> <p>Student constructs and communicates a complete response based on explanations/reasoning using the:</p> <ul style="list-style-type: none"> • Tools: <ul style="list-style-type: none"> ○ Manipulatives ○ Five Frame ○ Ten Frame ○ Number Line ○ Part-Part-Whole Model • Strategies: <ul style="list-style-type: none"> ○ Drawings ○ Counting All ○ Count On/Back ○ Skip Counting ○ Making Ten ○ Decomposing Number • Precise use of math vocabulary <p>Response includes a logical but incomplete progression of mathematical reasoning and understanding. Contains minor errors.</p>	<p>Student work shows partial understanding of the mathematics.</p> <p>Student constructs and communicates an incomplete response based on student’s attempts of explanations/ reasoning using the:</p> <ul style="list-style-type: none"> • Tools: <ul style="list-style-type: none"> ○ Manipulatives ○ Five Frame ○ Ten Frame ○ Number Line ○ Part-Part-Whole Model • Strategies: <ul style="list-style-type: none"> ○ Drawings ○ Counting All ○ Count On/Back ○ Skip Counting ○ Making Ten ○ Decomposing Number • Precise use of math vocabulary <p>Response includes an incomplete or illogical progression of mathematical reasoning and understanding.</p>	<p>Student work shows little understanding of the mathematics.</p> <p>Student attempts to construct and communicates a response using the:</p> <ul style="list-style-type: none"> • Tools: <ul style="list-style-type: none"> ○ Manipulatives ○ Five Frame ○ Ten Frame ○ Number Line ○ Part-Part-Whole Model • Strategies: <ul style="list-style-type: none"> ○ Drawings ○ Counting All ○ Count On/Back ○ Skip Counting ○ Making Ten ○ Decomposing Number • Precise use of math vocabulary <p>Response includes limited evidence of the progression of mathematical reasoning and understanding.</p>
5 points	4 points	3 points	2 points	1 point

DATA DRIVEN INSTRUCTION

Formative assessments inform instructional decisions. Taking inventories and assessments, observing reading and writing behaviors, studying work samples and listening to student talk are essential components of gathering data. When we take notes, ask questions in a student conference, lean in while a student is working or utilize a more formal assessment we are gathering data. Learning how to take the data and record it in a meaningful way is the beginning of the cycle.

Analysis of the data is an important step in the process. What is this data telling us? We must look for patterns, as well as compare the notes we have taken with work samples and other assessments. We need to decide what are the strengths and needs of individuals, small groups of students and the entire class. Sometimes it helps to work with others at your grade level to analyze the data.

Once we have analyzed our data and created our findings, it is time to make informed instructional decisions. These decisions are guided by the following questions:

- What mathematical practice(s) and strategies will I utilize to teach to these needs?
- What sort of grouping will allow for the best opportunity for the students to learn what it is I see as a need?
- Will I teach these strategies to the whole class, in a small guided group or in an individual conference?
- Which method and grouping will be the most effective and efficient? What specific objective(s) will I be teaching?

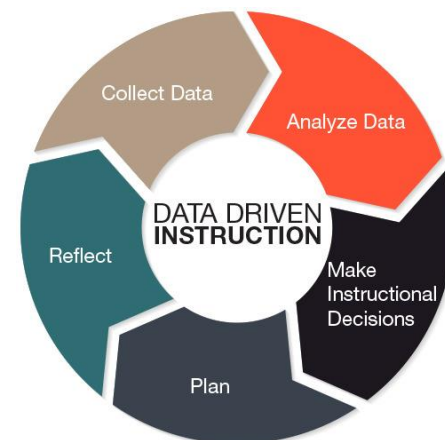
Answering these questions will help inform instructional decisions and will influence lesson planning.

Then we create our instructional plan for the unit/month/week/day and specific lessons.

It's important now to reflect on what you have taught.

Did you observe evidence of student learning through your checks for understanding, and through direct application in student work?

What did you hear and see students doing in their reading and writing?



Data Analysis Form School: _____ Teacher: _____ Date: _____

Assessment: _____ NJSL: _____

GROUPS (STUDENT INITIALS)	SUPPORT PLAN	PROGRESS
MASTERED (86% - 100%) (PLD 4/5):		
DEVELOPING (67% - 85%) (PLD 3):		
INSECURE (51%-65%) (PLD 2):		
BEGINNING (0%-50%) (PLD 1):		

MATH PORTFOLIO EXPECTATIONS

The Student Assessment Portfolios for Mathematics are used as a means of documenting and evaluating students' academic growth and development over time and in relation to the NJSL. The September task entry(-ies) should reflect the prior year content and *can serve* as an additional baseline measure.

All tasks contained within the **Student Assessment Portfolios** should be aligned to NJSL and be “practice forward” (closely aligned to the Standards for Mathematical Practice).

Four (4) or more additional tasks will be included in the **Student Assessment Portfolios** for Student Reflection and will be labeled as such.

K-2 GENERAL PORTFOLIO EXPECTATIONS:

- Tasks contained within the Student Assessment Portfolios are “practice forward” and denoted as “Individual”, “Partner/Group”, and “Individual w/Opportunity for Student Interviews¹”.
- Each Student Assessment Portfolio should contain a “Task Log” that documents all tasks, standards, and rubric scores aligned to the performance level descriptors (PLDs).
- Student work should be attached to a completed rubric; with appropriate teacher feedback on student work.
- Students will have multiple opportunities to revisit certain standards. Teachers will capture each additional opportunity “as a new and separate score” in the task log.
- A 2-pocket folder for each Student Assessment Portfolio is *recommended*.
- All Student Assessment Portfolio entries should be scored and recorded as an Authentic Assessment grade (25%)².
- All Student Assessment Portfolios must be clearly labeled, maintained for all students, inclusive of constructive teacher and student feedback and accessible for review.

GRADES K-2

Student Portfolio Review

Provide students the opportunity to review and evaluate their portfolio at various points throughout the year; celebrating their progress and possibly setting goals for future growth. During this process, students should retain ALL of their current artifacts in their Mathematics Portfolio

2nd Grade Authentic Assessment #1 – Counting Mice

Snake has 49 mice, and his cousin Rattles has 27 mice.

Part 1:

Who has more mice, Snake or Rattles?

Part 2:

How many more mice does he have? Show how you found the answer using words, numbers, and/or pictures

Part 3:

Snake and Rattles put their mice together in one big tank. 17 of the mice escape. How many mice do Snake and Rattles have now? Show how you found the answer using words, numbers, and/or pictures

Counting Mice				
Domain	Operations and Algebraic Thinking, Number and Operations in Base Ten			
Cluster	Represent and solve problems involving addition and subtraction. Use place value understanding and properties of operations to add and subtract.			
Standard(s)	<p>2.OA.1 Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.</p> <p>2.NBT.5 Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.</p>			
Solution	Part 1: Snake has more mice. Part 2: 22 mice Part 3: 59 mice			
Level 5: Distinguished Command	Level 4: Strong Command	Level 3: Moderate Command	Level 2: Partial Command	Level 1: No Command
<p>Student correctly answers and clearly constructs and communicates a complete response based on explanations/reasoning using :</p> <ul style="list-style-type: none"> • properties of operations • relationship between addition and subtraction relationship • strategies based on place value <p>Response includes an efficient and logical progression of steps.</p>	<p>Student correctly answers and clearly constructs and communicates a complete response with one minor calculation error based on explanations/reasoning using:</p> <ul style="list-style-type: none"> • properties of operations • relationship between addition and subtraction relationship • strategies based on place value <p>Response includes a logical progression of steps</p>	<p>Student answers, clearly constructs, and communicates a complete response with minor calculation errors based on explanations/reasoning using:</p> <ul style="list-style-type: none"> • properties of operations • relationship between addition and subtraction relationship • strategies based on place value <p>Response includes a logical but incomplete progression of steps. Minor calculation errors.</p>	<p>Student answers, clearly constructs, and communicates a complete response with major calculation errors and/or conceptual errors based on explanations/reasoning using:</p> <ul style="list-style-type: none"> • properties of operations • relationship between addition and subtraction relationship • strategies based on place value <p>Response includes an incomplete or illogical progression of steps.</p>	<p>The student shows no work or justification.</p>

Core Instructional and Supplemental Materials (K-5)

EUREKA MATH V. 2019
(GREAT MINDS)

GRADE	TEACHER RESOURCES	STUDENT RESOURCES
K (v. 2019.)	<ul style="list-style-type: none"> • Teacher Edition: Module 1-6 • Eureka Math Teacher Resource Pack • Eureka K-5 PD Toolkit 	<ul style="list-style-type: none"> • Learn Workbook Set: Module 1-6 • Succeed Workbook Set: Module 1-6 • Practice Workbook, Fluency: Module 1-6
1	<ul style="list-style-type: none"> • Teacher Edition: Module 1-6 • Eureka Math Teacher Resource Pack • Eureka K-5 PD Toolkit 	<ul style="list-style-type: none"> • Learn Workbook Set: Module 1-6 • Succeed Workbook Set: Module 1-6 • Practice Workbook, Fluency: Module 1-6
2	<ul style="list-style-type: none"> • Teacher Edition: Module 1-8 • Eureka Math Teacher Resource Pack • Eureka K-5 PD Toolkit 	<ul style="list-style-type: none"> • Learn Workbook Set: Module 1-8 • Succeed Workbook Set: Module 1-8 • Practice Workbook, Fluency: Module 1-8
3		
4	<ul style="list-style-type: none"> • Teacher Edition: Module 1-7 • Eureka Math Teacher Resource Pack • Eureka K-5 PD Toolkit 	<ul style="list-style-type: none"> • Learn Workbook Set: Module 1-7 • Succeed Workbook Set: Module 1-7 • Practice Workbook, Fluency: Module 1-7
5	<ul style="list-style-type: none"> • Teacher Edition: Module 1-7 • Eureka Math Teacher Resource Pack • Eureka K-5 PD Toolkit 	<ul style="list-style-type: none"> • Learn Workbook Set: Module 1-7 • Succeed Workbook Set: Module 1-7 • Practice Workbook, Fluency: Module 1-7
	<ul style="list-style-type: none"> • Teacher Edition: Module 1-6 • Eureka Math Teacher Resource Pack • Eureka K-5 PD Toolkit 	<ul style="list-style-type: none"> • Learn Workbook Set: Module 1-6 • Succeed Workbook Set: Module 1-6 • Practice Workbook, Fluency: Module 1-6

MATH IN FOCUS v. 2015
(HOUGHTON MIFFLIN HARCOURT)

GRADE	TEACHER RESOURCES	STUDENT RESOURCES
K	<ul style="list-style-type: none"> • Teacher Edition (A & B) • Implementation Guide • Assessment Package • Enrichment Bundle • Extra Practice Set • Teacher and Student Activity Cards • Home -to- School Connection Book • Online Teacher Technology Kit • Big Book Set • Online Interactive Whiteboard Lessons 	<ul style="list-style-type: none"> • Student Edition A – Pt. 1 • Student Edition A – Pt. 2 • Student Edition B – Pt. 1 • Student Edition B – Pt. 2 • Online Student Technology Kit
1	<ul style="list-style-type: none"> • Teacher Edition (A & B) • Implementation Guide • Assessment Package • Enrichment Bundle • Extra Practice Guide • Reteaching Guide • Home -to- School Connection Book • Online Teacher Technology Kit • Fact Fluency • Online Interactive Whiteboard Lessons 	<ul style="list-style-type: none"> • Student Texts (A & B) • Student Workbooks • Online Student Technology Kit • Student Interactivities
2-5	<ul style="list-style-type: none"> • Teacher Edition (A & B) • Implementation Guide • Assessment Package • Enrichment Bundle • Extra Practice Guide • Transition Guides • Reteaching Guide • Home -to- School Connection Book • Online Teacher Technology Kit • Fact Fluency • Online Interactive Whiteboard Lessons 	<ul style="list-style-type: none"> • Student Texts (A & B) • Student Workbooks • Online Student Technology Kit • Student Interactivities

Supplemental Resources

Great Minds

<https://greatminds.org/>

Embarc

<https://embarc.online/>

Engage NY

[http://www.engageny.org/video-library?f\[0\]=im_field_subject%3A19](http://www.engageny.org/video-library?f[0]=im_field_subject%3A19)

Common Core Tools

<http://commoncoretools.me/>

<http://www.ccsstoolbox.com/>

<http://www.achievethecore.org/steal-these-tools>

Achieve the Core

<http://achievethecore.org/dashboard/300/search/6/1/0/1/2/3/4/5/6/7/8/9/10/11/12>

Manipulatives

<http://nlvm.usu.edu/en/nav/vlibrary.html>

<http://www.explorelearning.com/index.cfm?method=cResource.dspBrowseCorrelations&v=s&id=USA-000>

<http://www.thinkingblocks.com/>

Illustrative Math Project :<http://illustrativemathematics.org/standards/k8>

Inside Mathematics: <http://www.insidemathematics.org/index.php/tools-for-teachers>

Sample Balance Math Tasks: <http://www.nottingham.ac.uk/~ttzedweb/MARS/tasks/>

Georgia Department of Education:<https://www.georgiastandards.org/Common-Core/Pages/Math-K-5.aspx>

Gates Foundations Tasks:<http://www.gatesfoundation.org/college-ready-education/Documents/supporting-instruction-cards-math.pdf>

References

“Eureka Math” *Great Minds*. 2018 < <https://greatminds.org/account/products>>